

# KHAGOL

## खगोल



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1st Issue  
ADITI - The First IUCAA building



10th Issue  
Charles Townes at IUCAA



15th Issue  
R. Balasubramanian explaining the concept of irrational numbers to school students



18th Issue  
IUCAA Director with his Guru, Fred Hoyle



20th Issue  
SAC members, watching the sky during "daytime" through the Automated Photoelectric Telescope, built at IUCAA



25th Issue  
R. A. Mashelkar delivering the 7th IUCAA Foundation Day Lecture.

*On the occasion of the 51st issue of Khagol, here is a bird's-eye-view of how IUCAA activities and Khagol evolved over the period 1990-2002.*

# 51



33rd Issue  
15th meeting of the International Society on General Relativity and Gravitation (GR - 15)



37th Issue  
Sympathetic Swings at the IUCAA Science Park



42nd Issue  
First prize winning entry of the drawing competition held during the National Science Day in February 2000



46th Issue  
Participants of the workshop on Astronomical Photometry & Spectroscopy



50th Issue  
Participants of the 21st Meeting of the ASI.

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### **Welcome to . . .**

**Banibrata Mukhopadhyay**, who has joined as a Post-doctoral Fellow. His reaserch interests are accretion disk, matter flows and inertial forces around compact objects, and behaviour of spin-half and spin-one particles in curved spacetime.

### **Congratulations**

We congratulate **Amalendu Bandopadhyay**, working at the M.P. Birla Institute of Fundamental Research for receiving the prestigious "Gopal Chandra Bhattacharya Memorial Award of the Government of West Bengal for Science Popularisation activities in the whole of West Bengal.

We congratulate **T. Padmanabhan** for receving the second prize for the year 2002 for his essay titled "*The Holography of Gravity Encoded in a Relation Between Entropy, Horizon Area and Action for Gravity*" given by Gravity Research Foundation, USA.

### **Acknowledgement**

This is the 51st issue of Khagol. This newsletter of IUCAA has grown and transformed itself along with IUCAA over the years and the first page of this issue gives you a glimpse of this metamorphosis. Starting this issue we hope to introduce new features, columns, etc. in order to make it more useful to the community which Khagol serves.

The response of the readers to Khagol has been extremely positive and we have also received several suggestions for newer features which we are currently considering. You will hear more about them in the coming issues. Please continue to let us have your reactions.

Khagol has benefitted tremendously from the efforts and support of the entire IUCAA staff over the years. In particular, the following people were directly involved with the production of Khagol at one time or the other (in alphabetical order): V. Chellathurai, Naresh Dadhich, Ajit Kembhavi, Santosh Khadilkar, Manjiri Mahabal, J. V. Narlikar, T. Padmanabhan, Arvind Paranjpye, late N. C. Rana, and S. N. Tandon.

**T. Padmanabhan**  
Editor

IUCAA is happy to announce the selection of the thirteenth batch of Visiting Associates. The Visiting Associateship is for a tenure of three years, beginning July 1, 2002.

### New Visiting Associates

- Rashmi Bhardwaj, Guru Gobind Singh Indrapratha Univesity, Delhi.
- Satyabrata Biswas, University of Kalyani.
- Tanuka Chatterjee (Kanjilal), Shibpur D.B. College, Howrah.
- Kalyan K. Mondal, Raja Peary Mohan College, Hooghly.
- Sanjay K. Pandey, L.B.S. College, Gonda.
- K.D. Patil, B.D. College of Engineering, Sevagram.
- Ninan Sajeeth Philip, St. Thomas College, Kozhencherry.
- S.K. Popalghat, J.E.S. College, Jalna.
- Anirudh Pradhan, Hindu P.G. College, Zamania.
- Rastogi Shantanu, D.D.U. Gorakhpur University.
- Saibal Ray, Barasat Government College.
- Ravindra V. Saraykar, Nagpur University.
- Bhim Prasad Sarmah, Tezpur University.
- T.R. Seshadri, University of Delhi.
- Rajendra N. Shelke, College of Engineering, Badnera.
- R.C. Verma, Punjabi University, Patiala.

### Extension to the tenth batch of Visiting Associates.

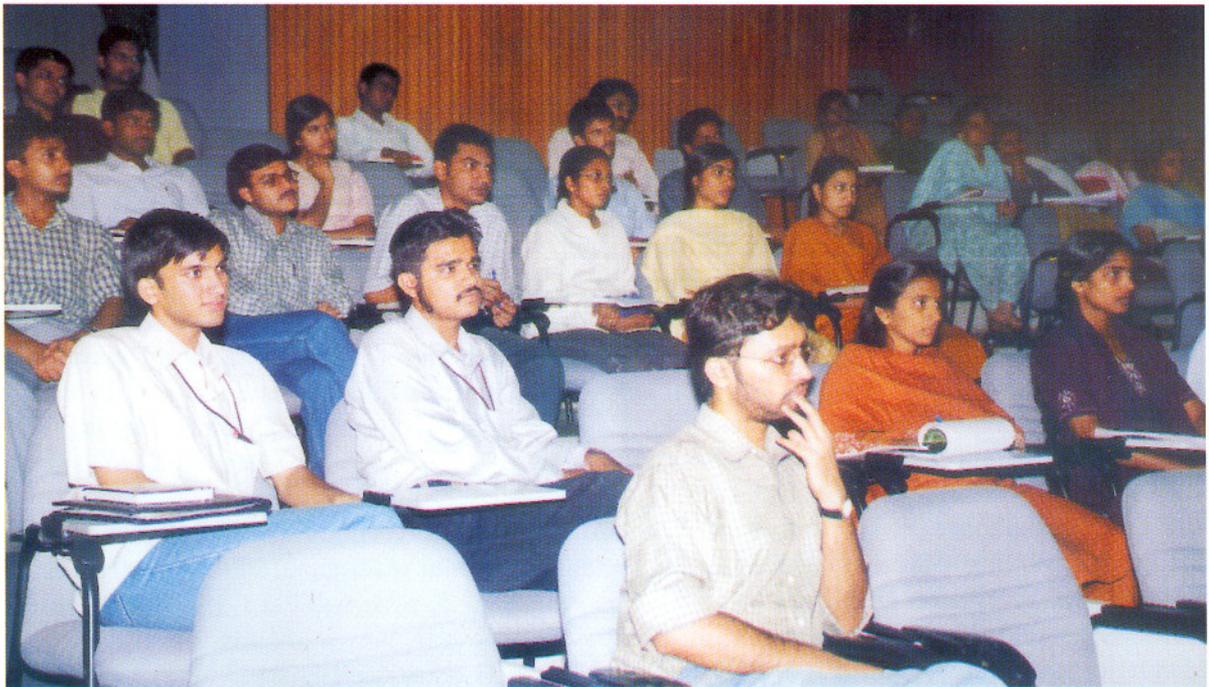
- Zafar Ahsan, Aligarh Muslim University.
- Bindu A. Bambah, University of Hyderabad.
- Asit Banerjee, Jadavpur University.
- S.P. Bhatnagar, Bhavnagar University.
- Somenath Chakrabarty, University of Kalyani.
- D.K. Chakraborty, Pt. Ravishankar Shukla University, Raipur.
- Deepak Chandra, S.G.T.B. Khalsa College, New Delhi.
- Suresh Chandra, Swami Ramanand Teerth Marathwada University, Nanded.
- Arnab Rai Choudhuri, Indian Institute of Science, Bangalore.
- Mrinal Kanti Das, Sri Venkateswara College, New Delhi.
- Jishnu Dey, Maulana Azad College, Kolkata.
- Mira Dey, Presidency College, Kolkata.
- Anil D. Gangal, University of Pune.
- Prem P. Hallan, Zakir Husain College, New Delhi.
- Syed N. Hasan, Osmania University, Hyderabad.
- K. Indulekha, Mahatma Gandhi University, Kottayam.
- Pushpa Khare, Utkal University, Bhubaneswar.
- Ashok C. Kumbharkhane, Swami Ramanand Teerth Marathwada University, Nanded.
- V.C. Kuriakose, Cochin University of Science and Technology, Kochi.
- Daksh Lohiya, University of Delhi.
- Usha Malik, Miranda House, Delhi.
- S. Mukherjee, University of North Bengal, Darjeeling.
- S.K. Pandey, Pt. Ravishankar Shukla University, Raipur.
- Lallan Prasad, M.B. Govt. P.G. College, Nainital.
- P. Vivekananda Rao, Osmania University, Hyderabad.
- Lal Mohan Saha, Zakir Husain College, New Delhi.
- Ramesh Tikekar, Sardar Patel University, Vallabh Vidyanagar.

## ***Introductory Summer School on Astronomy and Astrophysics***

A summer school for students of the B.Sc. final year and M.Sc. first year was organized jointly by IUCAA and National Centre for Radio Astrophysics (NCRA) at Pune during May 20 to June 21, 2002. The school is part of an annual series of Summer Schools on Astronomy and Astrophysics, sponsored by the Department of Science and Technology under which, the schools are conducted alternately at Bangalore and Pune. Lectures covering different theoretical, observational and instrumental aspects of astronomy and astrophysics were given by lecturers from IUCAA, NCRA and TIFR, Mumbai. The students did a reading assignment, under the supervision of a faculty member from IUCAA/NCRA and presented a ten minute talk. Students visited the GMRT site. Altogether 32 students from all over India attended the school. R. Misra from IUCAA and D. J. Saikia from NCRA were the school coordinators.



**Students and J. V. Narlikar engrossed in a discussion**



**Students at one of the lecture sessions**

## Gravitational Clustering in Static and Expanding Backgrounds

The statistical mechanics of systems dominated by gravity is of interest both from the theoretical and “practical” perspectives. Theoretically, this field has close connections with areas of condensed matter physics, fluid mechanics, renormalization group, etc. From the practical point of view, the ideas find application in different areas of astrophysics and cosmology, especially in the study of globular clusters, galaxies and gravitational clustering in the expanding universe. [For a review of statistical mechanics of gravitating systems in static background, see [1]; textbook descriptions are in [2]; gravitational clustering in cosmology is reviewed in [3] and in the textbooks [4]; for a sample of attempts by different groups see [5] and the references cited therein.]

### 1 Gravitational clustering in static background

To construct the statistical description of a system of  $N$  self gravitating point particles, one should begin with the construction of the micro canonical ensemble describing such a system. If  $g(E)$  is the volume of the constant energy surface  $H(p_i, q_i) = E$ , then the entropy and the temperature of the system will be  $S(E) = \ln g(E)$  and  $T(E) \equiv \beta(E)^{-1} = (\partial S / \partial E)^{-1}$ . (The finiteness of  $g$  requires the system to be confined to a finite volume in space for *any* system).

Systems for which a description based on canonical ensemble is possible, the Laplace transform of  $g(E)$  with respect to a variable  $\beta$  will give the partition function  $Z(\beta)$ . Gravitating systems of interest in astrophysics, however, cannot be described by a canonical ensemble [1], [6]. Virial theorem holds for such systems and we have  $(2K + U) = 0$ , where  $K$  and  $U$  are the total kinetic and potential energies of the system. This leads to  $E = K + U = -K$ ; since the temperature of the system is proportional to the total kinetic energy, the specific heat will be negative:  $C_V \equiv (\partial E / \partial T)_V \propto (\partial E / \partial K) < 0$ . On the other hand, the specific heat of any system described by a canonical ensemble  $C_V = \beta^2 \langle (\Delta E)^2 \rangle$  will be positive definite. Thus, one cannot describe self gravitating systems of the kind we are interested in by canonical ensemble.

One can, however, attempt to find the equilibrium configuration for self gravitating systems by maximizing the entropy  $S(E)$  or the phase volume  $g(E)$ . For a system of point particles, there is again no global maximum for entropy [1],[2]. If we move a small number of these particles arbitrarily close to each other, the potential energy of interaction of a pair of these particles,  $-Gm_1m_2/r_{12}$ , will become arbitrarily high as  $r_{12} \rightarrow 0$ . Transferring some of this energy to the rest of the particles, we can increase their kinetic energy without limit. This will clearly increase the phase volume occupied by the system (in the momentum space) without bound. This argument can be made more formal by dividing the original system into a small, compact core, and a large diffuse halo and allowing the core to collapse and transfer the energy to the halo.

The absence of the global maximum for entropy - as argued above - depends on the lack of small scale cutoff. If we assume, instead, that each particle has a radius  $a$ , there will be an upper bound on the amount of energy that can be made available to the rest of the system. Further, no real system is completely isolated and to obtain a truly isolated system, we need to confine the system inside a spherical region of radius  $R$  with, say, reflecting wall.

The two cut-offs  $a$  and  $R$  will make the upper bound on the entropy finite, but even with the two cut-offs the formation of a compact core and a diffuse halo will still occur, since this is the direction of increasing entropy. Particles in the hot diffuse component will permeate the entire spherical cavity, bouncing off the walls and having a kinetic energy which is significantly larger than the potential energy. The compact core will exist as a gravitationally bound system with very little kinetic energy. A formal way of understanding this phenomena is based on the virial theorem [2]:

$$2T + U = 3PV + \Phi \quad (1)$$

for a system with a short distance cut-off confined to a sphere of volume  $V$ , where  $P$  is the pressure on the walls and  $\Phi$  is the correction to the potential energy arising from the short distance cut-off. This equation can be satisfied in essentially three different ways. If  $T$  and  $U$  are significantly higher than  $3PV$  and  $\Phi$ , then we have  $2T + U \approx 0$  which describes a self gravitating systems in standard virial equilibrium, but not in the state of maximum entropy. If  $T \gg U$  and  $3PV \gg \Phi$ , one can have  $2T \approx 3PV$  which describes an ideal gas with no potential energy confined to a container of volume  $V$ ; this will describe the hot diffuse component at late times. If  $T \ll U$  and  $3PV \ll \Phi$ , then one can have  $U \approx \Phi$ , describing the compact potential energy dominated core at late times. Such an asymptotic state with two distinct phases is quite different from what would have been expected for systems with only short range interaction. If the gravitating system is put in a heat bath and the temperature is varied, a sudden phase transition occurs at a critical temperature, leading to the formation of the two phases [7], [1].

There are, however, configurations which are *local extrema* of entropy, which are not global maxima. Intuitively, one would have expected the distribution of matter in such configuration to be described by a Boltzmann distribution, with the  $\rho(\mathbf{x}) \propto \exp[-\beta\phi(\mathbf{x})]$ , where  $\phi$  is the gravitational potential related to the density  $\rho$  by Poisson equation. This configuration, called isothermal sphere, has a density profile  $\rho \propto x^{-2}$  asymptotically. Isothermal spheres with total energy  $E_i$  and mass  $M$ , however, cannot exist [8] if  $(RE/GM^2) < -0.335$ . Even when  $(RE/GM^2) > -0.335$ , the isothermal solution need not be stable. The stability of this solution can be investigated by studying the second variation of the entropy. Such a detailed analysis shows that the following results are true: (i) Systems with  $(RE/GM^2) < -0.335$  cannot evolve into isothermal spheres. Entropy has no extremum for such systems [1], [8]. (ii) Systems with  $((RE/GM^2) > -0.335)$

and ( $\rho(0) > 709 \rho(R)$ ) can exist in a meta-stable (saddle point state) isothermal sphere configuration. Here  $\rho(0)$  and  $\rho(R)$  denote the densities at the center and edge respectively. The entropy extrema exist but they are not local maxima. (iii) Systems with  $((RE/GM^2) > -0.335)$  and ( $\rho(0) < 709 \rho(R)$ ) can form isothermal spheres, which are local maximum of entropy. These are striking peculiarities in the case of SMGS and seem to find application in the physics of globular clusters.

## 2 Gravitational clustering in an expanding background

There is considerable amount of observational evidence to suggest that one of the dominant energy densities in the universe is contributed by self gravitating (nearly) point particles. The smooth average energy density of these particles drive the expansion of the universe while any small deviation from the homogeneous energy density will cluster gravitationally. It is often enough (and necessary) to use a statistical description and relate different statistical indicators (like the power spectra,  $n$ th order correlation functions, ...) of the resulting density distribution to the statistical parameters (usually the power spectrum) of the initial distribution.

The relevant scales at which gravitational clustering is nonlinear are less than about 10 Mpc, while the expansion of the universe has a characteristic scale of about 4000 Mpc [4]. Hence, nonlinear gravitational clustering in an expanding universe can be adequately described by Newtonian gravity by introducing a *proper* coordinate for the  $i$ -th particle  $\mathbf{r}_i$ , related to the *comoving* coordinate  $\mathbf{x}_i$ , by  $\mathbf{r}_i = a(t)\mathbf{x}_i$  where  $a(t)$  is the expansion factor. The Newtonian dynamics works with the proper coordinates  $\mathbf{r}_i$  which can be translated to the behaviour of the comoving coordinate  $\mathbf{x}_i$  by this rescaling.

If  $\mathbf{x}(t, \mathbf{q})$  is the position of a particle at time  $t$  with its initial position being  $\mathbf{q}$ , then equations for gravitational clustering in an expanding universe, in the Newtonian limit, can be summarized as [10], [3].

$$\ddot{\mathbf{x}} + \frac{2\dot{a}}{a}\dot{\mathbf{x}} = -\frac{1}{a^2}\nabla_{\mathbf{x}}\phi; \quad (2)$$

$$\begin{aligned} \ddot{\phi}_{\mathbf{k}} + 4\frac{\dot{a}}{a}\dot{\phi}_{\mathbf{k}} &= -\frac{1}{2a^2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\phi_{\frac{\mathbf{k}}{2}+\mathbf{p}}\phi_{\frac{\mathbf{k}}{2}-\mathbf{p}}\mathcal{G}(\mathbf{k}, \mathbf{p}) \\ &+ \left(\frac{3H_0^2}{2}\right)\int \frac{d^3\mathbf{q}}{a}\left(\frac{\mathbf{k}\cdot\dot{\mathbf{x}}}{k}\right)^2 e^{i\mathbf{k}\cdot\mathbf{x}}, \end{aligned} \quad (3)$$

where  $\mathbf{x} = \mathbf{x}(t, \mathbf{q})$ ,  $\mathcal{G}(\mathbf{k}, \mathbf{p}) = (k/2)^2 + p^2 - 2(\mathbf{k}\cdot\mathbf{p}/k)^2$  and  $\phi_{\mathbf{k}}(t)$  is the Fourier transform of the gravitational potential  $\phi(t, \mathbf{x})$  due to perturbed density.

Equation (3) is exact but involves  $\dot{\mathbf{x}}(t, \mathbf{q})$  on the right hand side and hence, cannot be considered as closed. Together, the two equations form a closed set but solving them exactly

is an impossible task. It is, however, possible to use this equation with several well motivated approximations [9] to obtain information about the system. I shall briefly mention a few of them.

Consider first the effect of a bunch of particles, in a virialized cluster, on the rest of the system. This is described, to the lowest order, by just the monopole moment of the cluster – which can be taken into account by replacing the cluster by a single particle at the centre of mass having appropriate mass. Such a replacement should not affect the evolution at scales much bigger than the cluster size. At first sight, one may wonder how this feature (“renormalizability of gravity”) is taken care of in equation (3). Inside a galaxy cluster, for example, the velocities  $\dot{\mathbf{x}}$  can be quite high and one might wonder whether this could influence the evolution of  $\phi_{\mathbf{k}}$  at all scales. This does not happen and, to the lowest order, the contribution from virialized bound clusters cancel [9], [10] in the two terms in the right hand side of (3).

In this limit,  $\phi_{\mathbf{k}}$  is constant in time and the Poisson equation  $-k^2\phi_{\mathbf{k}} = 4\pi G\rho_b a^2\delta_{\mathbf{k}} \propto (\delta_{\mathbf{k}}/a)$  in the matter dominated universe with  $a(t) \propto t^{2/3}$ ,  $\rho_b \propto a^{-3}$  implies that the density contrast has the growing solution  $\delta_{\mathbf{k}}(t) = [a(t)/a(t_i)]\delta_{\mathbf{k}}(t_i)$ . The power spectrum  $P(\mathbf{k}, t) = \langle |\delta_{\mathbf{k}}(t)|^2 \rangle$  and the correlation function  $\xi(\mathbf{x}, t)$  [which is the Fourier transform of the power spectrum] both grow as  $a^2(t)$ . This allows us to fix the evolution of clustering at sufficiently large scales uniquely. The clustering at these scales, which is well described by linear theory, grows as  $a^2$ .

There is, however, an important caveat to this claim. While ignoring the right hand side of (3) one is comparing its contribution at any wave number  $\mathbf{k}$  to the contribution in linear theory. If at the relevant wavenumber, the contribution from linear evolution is negligibly small, then the *only* contribution will come from the terms on the right hand side and, of course, we cannot ignore it in this case. This contribution will scale as  $k^2 R^2$ , where  $R$  is the typical scale of virialized systems and will lead to a development of  $\delta_{\mathbf{k}} \propto k^2$ ,  $P(k) \propto k^4$  at small  $k$ . Thus, if the large scales have too little power intrinsically (i.e., if  $n > 4$ ), then the long wavelength power will soon be dominated by the “ $k^4$  - tail” of the short wavelength power arising from the nonlinear clustering. This is an interesting and curious result which is characteristic of gravitational clustering.

## 3 Nonlinear scaling relations

As to be expected, cosmological expansion completely changes the asymptotic nature of the problem. The problem has now become time dependent and it will be pointless to look for “equilibrium solutions”.

There are three key theoretical questions which are of considerable interest in this area which I will briefly summarise:

- If the initial power spectrum is sharply peaked in a narrow band of wavelengths, how does the evolution trans-

for the power to other scales? (This is, in some sense, analogous to determining the Green function for the gravitational clustering except that superposition will not work in the nonlinear context.)

- Do the virialized structures formed in an expanding universe due to gravitational clustering have any invariant properties? Can their structure be understood from first principles?
- Does the gravitational clustering at late stages wipe out the memory of initial conditions or does the late stage evolution depend on the initial power spectrum of fluctuations?

To make any progress with these questions we need a robust prescription which will relate statistical indicators like the two-point correlation function in the nonlinear regime to the initial power spectrum. Fortunately, this problem has been solved [11] to a large extent and hence, one can use this as a basis for attacking these questions.

The nonlinear mean correlation function can be expressed in terms of the linear mean correlation function by the relation:

$$\bar{\xi}(a, x) = \begin{cases} \bar{\xi}_L(a, l) & (\text{for } \bar{\xi} < 1) \\ \bar{\xi}_L(a, l)^D & (\text{for } 1 < \bar{\xi} < 125) \\ 11.7\bar{\xi}_L(a, l)^{Dh/2} & (\text{for } 125 < \bar{\xi}) \end{cases} \quad (4)$$

where  $l = x[1 + \bar{\xi}(a, x)]^{1/D}$ ,  $D = 2, 3$  is the dimension of space and  $h$  is a constant. [The results of numerical simulation in 2D, suggests that  $h = 3/4$  asymptotically. We will discuss the 3D results in more detail below]. The numerical values are for  $D = 3$ .

One could use this to examine whether the power spectrum (or the correlation function) has a universal shape at late times, independent of initial power spectrum. This is indeed true [12] if the initial spectrum was sharply peaked. In this case, at length scales smaller than the initial scale at which the power is injected, the two point correlation function has a universal asymptotic shape of  $\bar{\xi}(a, x) \propto a^2 x^{-1} (L+x)^{-1}$ , where  $L$  is the length scale at which  $\bar{\xi} \approx 200$ . This can be understood as follows:

In the quasi-linear phase, regions of high density contrast will undergo collapse and in the nonlinear phase more and more virialized systems will get formed. We recall that, in the study of finite gravitating systems made of point particles and interacting via Newtonian gravity, isothermal spheres play an important role and are the local maxima of entropy. Hence, dynamical evolution drives the system towards an  $(1/x^2)$  profile. Since, one expects similar considerations to hold at small scales, during the late stages of evolution of the universe, we may hope that isothermal spheres with  $(1/x^2)$  profile may still play a role in the late stages of evolution of clustering in an expanding background. However, while converting the density profile to correlation function, we need to distinguish

between two cases. In the quasi-linear regime, dominated by the collapse of high density peaks, the density profile around any peak will scale as the correlation function and we will have  $\bar{\xi} \propto (1/x^2)$ . On the other hand, in the nonlinear end, we will be probing the structure inside a single halo and  $\xi(\mathbf{x})$  will vary as  $\langle \rho(\mathbf{x} + \mathbf{y})\rho(\mathbf{y}) \rangle$ . If  $\rho \propto |x|^{-\epsilon}$ , then  $\bar{\xi} \propto |x|^{-\gamma}$  with  $\gamma = 2\epsilon - 3$ . This gives  $\bar{\xi} \propto (1/x)$  for  $\epsilon = 2$ . Thus, if isothermal spheres are the generic contributors, then we expect the correlation function to vary as  $(1/x)$  and nonlinear scales, steepening to  $(1/x^2)$  at intermediate scales. Further, since isothermal spheres are local maxima of entropy, a configuration like this should remain undistorted for a long duration. This argument suggests that a  $\bar{\xi}$  which goes as  $(1/x)$  at small scales and  $(1/x^2)$  at intermediate scales is likely to grow approximately as  $a^2$  at all scales. At scales bigger than the scale at which power was originally injected, the spectrum develops a  $k^4$  tail for reasons described before. This is confirmed by simulations for sharply peaked initial spectra. But if the initial spectrum is *not* sharply peaked, each band of power evolves by this rule and the final result is a lot messier.

The second question one could ask, concerns the density profiles of individual virialized halos. If the density field  $\rho(a, \mathbf{x})$  at late stages can be expressed as a superposition of several halos, each with some density profile  $f(\mathbf{x})$  then the  $i$ -th halo centred at  $\mathbf{x}_i$  will contribute a density  $f(\mathbf{x} - \mathbf{x}_i, a)$  at the location  $\mathbf{x}$ . The power spectrum for the density contrast,  $\delta(a, \mathbf{x}) = (\rho/\rho_b - 1)$ , will be  $P(k) = |f(k)|^2 P_c(k)$ , where  $P_c(\mathbf{k}, a)$  denotes the power spectrum of the distribution of centers of the halos. If the correlation function  $\bar{\xi} \propto x^{-\gamma}$ , the correlation function of the centres  $\bar{\xi} \propto x^{-\gamma_c}$  and the individual profiles are of the form  $f(x) \propto x^{-\epsilon}$ , then this relation translates to  $\epsilon = 3 + (1/2)(\gamma - \gamma_c)$ .

At very nonlinear scales, the centres of the virialized clusters will coincide with the deep minima of the gravitational potential. Hence, the power spectrum of the centres will be proportional to the power spectrum of the gravitational potential  $P_\phi(k) \propto k^{n-4}$  if  $P(k) \propto k^n$ . Since the correlation functions vary as  $x^{-(\alpha+3)}$  when the power spectra vary as  $k^\alpha$ , it follows that  $\gamma = \gamma_c - 4$ . Substituting into the above relation, we find that  $\epsilon = 1$  at the extreme nonlinear scales. On the other hand, in the quasi-linear regime, reasonably large density regions will act as cluster centres and hence, one would expect  $P_c(k)$  and  $P(k)$  to scale in a similar fashion. This leads to  $\gamma \approx \gamma_c$ , giving  $\epsilon \approx 3$ . So we would expect the halo profile to vary as  $x^{-1}$  at small scales steepening to  $x^{-3}$  at large scales. A simple interpolation for such a density profile will be

$$f(x) \propto \frac{1}{x(x+l)^2}. \quad (5)$$

Such a profile, usually called NFW profile [13], is often used in cosmology. The argument given above, however, is very tentative and it is difficult to obtain (5) from a more rigorous theoretical analysis.

In fact, it is possible to reach different conclusions regarding the asymptotic evolution of the system from differ-

ent physical assumptions [14]. The NSR in (4) for 3-D with constant  $h$  leads to the asymptotic correlation function

$$\bar{\xi}(a, x) \propto a^{\frac{2\gamma}{n+3}} x^{-\gamma}; \quad \gamma = \frac{3h(n+3)}{2+h(n+3)} \quad (6)$$

for an initial spectrum which is scale-free power law with index  $n$ . If we assume that the evolution gets frozen in proper coordinates at highly nonlinear scales then it is easy to show that  $h = 1$ . If this assumption (called stable clustering) is valid, then the late time behaviour of  $\bar{\xi}(a, x)$  is strongly dependent on the initial conditions and (6) shows that  $\bar{\xi}(a, x)$  at nonlinear scales will be as,

$$\bar{\xi}(a, x) \propto a^{\frac{6}{n+5}} x^{-\frac{3(n+3)}{n+5}}; \quad (\bar{\xi} \gg 200). \quad (7)$$

In other words the two (apparently reasonable) requirements: (i) validity of stable clustering at highly nonlinear scales and (ii) the independence of late time behaviour from initial conditions, are *mutually exclusive*. [At present, there exists some evidence from simulations [15] that this process, called stable clustering, does *not* occur in the  $a \propto t^{2/3}$  cosmological model; but this result is not definitive].

In the very nonlinear limit, the correlation function probes the interiors of individual halos and we have  $\epsilon = (1/2)(3+\gamma)$ . [This corresponds to  $P_c = \text{constant}$ ,  $\gamma_C = 3$  in the previous discussion.] If  $\gamma$  depends on  $n$  so will  $\epsilon$  and the individual halos will remember the initial power spectrum.

We can obtain a  $\gamma$  which is independent of initial power law index provided  $h$  satisfies the condition  $h(n+3) = c$ , a constant. In this case, the halo profile will be given by  $\epsilon = 3(c+1)/(c+2)$ . Note that we are now demanding the asymptotic value of  $h$  to *explicitly depend* on the initial conditions though the *spatial* dependence of  $\bar{\xi}(a, x)$  does not. As an example of the power of such a — seemingly simple — analysis, note the following: Since  $c \geq 0$ , it follows that  $\epsilon > (3/2)$ ; invariant profiles with shallower indices (for e.g with  $\epsilon = 1$ ) discussed above are not consistent with the evolution described above. One requires very high resolution simulations to verify the condition  $h(n+3) = c$  and the current results are inconclusive.

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Listed below are the IUCAA preprints released during April - June 2002. These can be obtained from the Librarian, IUCAA (library@iucaa.ernet.in).

S. V. Dhurandhar, *Gravitational wave astronomy: Recent advances*, IUCAA-11/2002; T. Padmanabhan, *Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes*, IUCAA-12/2002; T. Padmanabhan, *Why do we observe a small but non-zero cosmological constant?*, IUCAA-13/2002; S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, *Hawking radiation in different coordinate settings: Complex paths approach*, IUCAA-14/2002; Joydeep Bagchi, et.al., *Evidence for shock acceleration and intergalactic magnetic fields in a large-scale filament of galaxies ZwC1 2341.1+0000*, IUCAA-15/2002; T. Padmanabhan, *Accelerated expansion of the universe driven by tachyonic matter*, IUCAA-16/2002; T. Padmanabhan and T. Roy Choudhury, *Can the clustered dark matter and smooth dark energy arise from the same scalar field?*, IUCAA-17/2002; S. G. Ghosh and Naresh Dadhich, *Gravitational collapse of type II fluid in higher dimensional space-times*, IUCAA-18/2002; Yuri Shtanov and Varun Sahni, *New cosmological singularities in braneworld models*, IUCAA-19/2002; R. G. Vishwakarma, *A Machian*

*model of dark energy*, IUCAA-20/2002; M. Sami, *Implementing power law inflation with rolling tachyon on the brane*, IUCAA-21/2002; T. Padmanabhan, *The holography of gravity encoded in a relation between entropy, horizon area and action for gravity*, IUCAA-22/2002; T. Padmanabhan, *Is gravity an intrinsically quantum phenomenon? Dynamics of gravity from the entropy of spacetime and the principle of equivalence*, IUCAA-23/2002; M. Sami, P. Chingangbam and T. Qureshi, *Aspects of Tachyonic inflation with exponential potential*, IUCAA-24/2002; Tapas K. Das, *Generalized shock solutions for hydrodynamic black hole accretion*, IUCAA-25/2002; D.V. Ahluwalia, N. Dadhich and M. Kirchbach, *On the spin of gravitational bosons*, IUCAA-26/2002; P.S. Joshi, N. Dadhich and Roy Maartens, *Why do naked singularities form in gravitational collapse?*, IUCAA-27/2002; T. Padmanabhan, *Statistical mechanics of gravitating systems in static and cosmological backgrounds*, IUCAA-28/2002; R. Srianand, et.al., *A collimated flow driven by radiative pressure from the nucleus of quasar Q 1511+091*, IUCAA-29/2002; C. Ledoux, R. Srianand and P. Petitjean, *Detection of molecular hydrogen in a near Solar-metallicity damped Lyman -  $\alpha$  system at  $Z_{abc} = 2$  toward Q 0551 - 366*, IUCAA-30/2002.

### Seminars

04.04.2002 Sukratu Barve on *Quantum stress tensor on Cauchy horizons*; 13.05.2002 B. J. Ahmedov on *Electromagnetic fields of magnetized neutron stars in general relativity*; 16.05.2002 Ninan Sajeeth Philip on *An introduction to machine learning tools*; 30.05.2002 K. Indulekha on *Formation of bound open clusters in galaxies*; 31.05.2002 Subenoy Chakraborty on *Asymptotic behaviour of homogeneous cosmological models*; 06.06.2002 Sandeep Sahijpal on *Our Solar system: The origin and the last nucleosynthesis(?)*; 13.06.2002 V. K. Gupta on *Radial oscillations of hybrid stars*; 17.06.2002 Pushpa Khare on *Gravitational lensing and the CDM model of the universe*; 18.06.2002 A. Banerjee on *Fluid spheres with heat flux and the junction conditions* and 19.6.2002 T. R. Seshadri on *Nature of galaxy clustering and the transition to homogeneous distribution*.

### Colloquium

10.06.2002 K. P. Singh on *Telescopes for X-ray astronomy*.

## School Students' Summer Programme

This year's School Students' Summer Programme was held from April 15 to May 24, 2002. IUCAA has been conducting this programme for the students of VIII and IX standards since 1993, to give them a brief insight of doing scientific research. The programme is open to the students in Pune.

Each week a new batch of 30 students was invited to work in projects at IUCAA from Monday to Friday. Groups of four to six students were attached to each individual guide. The programme has no set syllabus or course guidelines. The students and the guide work out their own schedule for the week. The students were given access to the IUCAA library.

On their very first day at IUCAA, the students were briefed about the programme and soon after that they would go with their guides to work on different projects. During the week they also participated in various common activities. Vinaya Kulkarni gave them guided tour of the science park and they participated in operating the internet telescope at Mt. Wilson, California, the facility kindly provided by the Gilbert Clark, the Director, Telescope in Education. Arvind Paranjpye carried out a general question answer session. On the last day of the programme, every student submitted his or her report on the work carried out during the week. The programme ended with an oral presentation by at least one student from every group followed by Jayant Narlikar's three 'mathematical teasers'.

A. L. Ahuja, V. Chellathurai, S. V. Dhurandhar, Ranjan Gupta, Amir Hajian, Harvinder Kaur Jassal, Vinaya Kulkarni, Sanjit Mitra, J. V. Narlikar, T. Padmanabhan, Arvind Paranjpye, A. N. Ramaprakash, Arvind Chandrakant Ranade, A. A. Sengupta, Jatush Sheth, Tarun Souradeep, Prasad Subramanian and S. N. Tandon were the guides. The projects included: Foucault pendulum, coriolis force and centrifugal force, Solar system, calculating diameters of planets, gravitational acceleration, gravitational force of planets, orbital period, Kepler's law of planetary motion, study of the Sun, the

Earth and the Moon, and tides in the ocean. Some of the students made periscopes and studied telescopes and usage of CCD in astronomy.

Some of them studied mathematical topics like modulo arithmetic, geometric progression, tricks to remember complex mathematical formulae, mathematical systems, Euler's functions, Fermat theorem, permutations and combinations, Bond percolation on a square lattice, and Random walk on square lattice. Some others worked out problem of water distribution for village of certain population. The problem included study of Archimedes principle, how siphon works, consumption of water, design and height of a tank for a colony of certain number of houses and designing a pump to deliver water to the overhead tank. A group of students also studied variation in temperature inside and outside of a brick house model made by them.

In their report, the students were also asked to give their impressions on the programme. To almost every one 'Coming to IUCAA was like a dream come true'. One student wrote that she "was not quite interested in mathematics" but now she enjoys it. A few students strongly suggested that since they never had access to actual sky watching, at least a one night programme may be conducted during their winter holidays. We have taken this suggestion very positively.

See the adjoining page for some of the glimpses of this programme.

The programme was coordinated by Arvind Paranjpye.

## *Some glimpses of the School Students' Summer Programme*



Students and S.V. Dhurandhar engrossed in a discussion



S.N. Tandon and the students at a practical session



Informal learning sessions formed part of this programme - H.K. Jassal with students

### *Visitors during April to June*

N. Druguet, S. Chandra, Nagendra Kumar, H. Sikka, S. Barway, J. Lasue, P. Vinu, K. Shanthi, B. Basu, Ravi Kulkarni, S. Chatterjee, M.L. Kurtadikar, P.M. Kokne, P.K. Suresh, D. Lohiya, A.K. Mittal, C. Jog, Ninan S. Philip, N. Bhat, S. Bhattacharya, A. Nigavekar, A. Abraham, C.D. Ravikumar, T. Chatterjee, K.P. Harikrishnan, M.C. Sabu, M.K. Patil, K. George, V.C. Kuriakose, A.C. Kumbharkhane, D.W. Deshkar, S.G. Ghosh, H.S. Das, M. Sami, S. Chakraborty, U. Debnath, K. Jotania, Rajshree Jotania, S. Sahijpal, M. Khan, D.C. Srivastava, K. Indulekha, G.V. Vijayagovindan, A. Banerjee, Y. Mathur, S. Singh, R. Kochar, R. Sahay, I.K. Mukherjee, S. K. Ray, B. Dasgupta, V.K. Gupta, A.A. Usmani, T.R. Seshadri, B. Ishwar, B.K. Sinha, L.K. Jha, K.N. Iyer, Lalan Prasad Verma, N.K. Lohani, P.K. Srivastava, P. Khare, V.O. Thomas, B.C. Paul, J. Magri, J. Grain, U. Goswami, A. Pradhan, S.S. Prasad, K. Shanthi and S.K. Pandey.

Thirty-two students visited IUCAA as participants of the Introductory Summer School in Astronomy and Astrophysics.

From this 51st issue, we are starting a new column discussing some interesting problems in physics / astrophysics. Each issue will carry 2-3 questions for which the answers will be given in the subsequent issue. The problems will range from idle curiosities to open-ended ones which require significant amount of physical insight. Here are two questions to set the ball rolling:

(1) (a) A collection of  $2n$  particles, each of mass  $m$ , are located at the vertices of a regular polygon with a circumscribing circle of radius  $R$ . At  $t = 0$  these particles start moving under their mutual gravitational attraction. Describe their subsequent motion. (b) In the limit of  $n \rightarrow \infty, m \rightarrow 0$  with the total mass  $2nm$  of the particles remaining finite, the problem reduces to that of a thin uniform circular ring collapsing under its own weight. What is the gravitational force at any one point on the ring due to the rest of the matter in the ring?

(2) An hour-glass is made of two identical conical shaped figures with their vertices connected together by a small orifice. The sand pours from the upper half to the lower half. Model this system and make an estimate of the time it takes for upper half to empty itself in terms of various parameters of the problem.

### **Visitors Expected**

#### **July**

S. Bhowmick, Barasat Government College; M. Sinha, Presidency College; J. Dey, Maulana Azad College, Calcutta; J. Bagla, Harish Chandra Research Institute, Allahabad; Nagendra Kumar, K.G.K. (PG) College, Moradabad.

#### **August**

M. Wainwright, University of Sheffield; N.C. Wickramasinghe, Cardiff University; David Lloyd, Cardiff University; Pushpa Bhargava, Anveshna, Hyderabad; S. Shivaji, Centre for Cellular and Molecular Biology, Hyderabad; G.S.N. Reddy, Centre for Cellular and Molecular Biology, Hyderabad; P. Rajaratnam, ISRO, Bangalore; S. Ramadurai, Tata Institute of Fundamental Research, Mumbai, P. Bak, University of Nariobi; D. Rosario, University of Virginia

#### **September**

R. Banerjee, Max Planck Institut for Astrophysics, Munich; K.S.V.S. Narasimhan, Chennai.

**Please note the change in Telephone and Fax numbers !!!**

***Khagol (the Celestial Sphere) is the quarterly bulletin of IUCAA. We welcome your responses at the following address:***

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